1. 1

2. $\ln(\sqrt{2}+1)$

3. $\dfrac{\ln(\ln x)+1}{x}(\ln x)^{\ln x}$

4. $1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}$

5. 정의역$=\{x:\ e^{-1}\le x\le e\}=[e^{-1},e]$

$y'=\dfrac{1}{x\sqrt{1-(\ln x)^2}}$

6. $\sqrt{2}$

7. $\dfrac{x}{\sqrt{1+x^2}}+C \quad =\sin(\tan^{-1}x)+C$

8. $\dfrac{\pi}{3\sqrt{3}}$

9. $2\sqrt{x}\,e^{\sqrt{x}}-2e^{\sqrt{x}}+C \quad =2e^{\sqrt{x}}(\sqrt{x}-1)+C$

10. $3x^{1/3}-3\tan^{-1}(x^{1/3})+C$

11. 만약 $|x| < 1$ 이면, $\sin^{-1}x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ 임을 보여라.

: $\sin^{-1}x = \alpha \ (-\frac{\pi}{2} < \alpha < \frac{\pi}{2})$ 라 두면 $\tan\alpha = \frac{x}{\sqrt{1-x^2}}$ 이므로

$\sin^{-1}x = \alpha = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ 이다.

12. 곡선 $y=\cosh x$ $(0 \le x \le 1)$를 $x-$축으로 회전시켜 얻은 곡면의 넓이를 구하여라.

: $y = \cosh x = \dfrac{e^x + e^{-x}}{2}$

곡면의 넓이= $\displaystyle\int_0^1 2\pi y\sqrt{1+(y')^2}\,dx = \int_0^1 2\pi \frac{(e^x + e^{-x})^2}{4}\,dx$

$$= \frac{\pi(e^2 - e^{-2} + 4)}{4}$$

13. $\displaystyle\int \frac{2x^2+5x+6}{x^2+2x+2}\,dx$ 를 구하여라.

: $\displaystyle\int \frac{2x^2+5x+6}{x^2+2x+2}\,dx = \int \left\{2+\frac{x+1}{x^2+2x+2}+\frac{1}{x^2+2x+2}\right\}dx$

$$= \int 2\,dx + \int \frac{x+1}{x^2+2x+2}\,dx + \int \frac{1}{(x+1)^2+1}\,dx = 2x + \frac{1}{2}\ln(x^2+2x+2) + \tan^{-1}(x+1) + C$$

14. $\displaystyle\int_1^e \frac{\ln x}{x\sqrt{1-(\ln x)^2}}\,dx$ 를 구하여라.

: $\ln x = u$로 치환하면

$$\int_1^e \frac{\ln x}{x\sqrt{1-(\ln x)^2}}\,dx = \int_0^1 \frac{u}{\sqrt{1-u^2}}\,du$$

$u = \sin\theta$로 치환하면

$$\int_1^e \frac{\ln x}{x\sqrt{1-(\ln x)^2}}\,dx = \int_0^1 \frac{u}{\sqrt{1-u^2}}\,du = \int_0^{\frac{\pi}{2}} \sin\theta\,d\theta = 1$$

15. 특이적분 $\int_{-1}^{4} \frac{1}{\sqrt{|x|}}\,dx$ 수렴, 발산을 조사하고 수렴하면 그 값을 구하여라.

: $\int_{-1}^{4} \frac{1}{\sqrt{|x|}}\,dx = \int_{-1}^{0} \frac{1}{\sqrt{|x|}}\,dx + \int_{0}^{4} \frac{1}{\sqrt{|x|}}\,dx$

$$\int_{-1}^{0} \frac{1}{\sqrt{|x|}}\,dx = \lim_{t \to 0^-} \int_{-1}^{t} \frac{1}{(-x)^{1/2}}\,dx = \lim_{t \to 0^-} \left[-2(-x)^{1/2}\right]_{-1}^{t} = 2$$

$$\int_{0}^{4} \frac{1}{\sqrt{|x|}}\,dx = \lim_{t \to 0^+} \int_{t}^{4} \frac{1}{\sqrt{x}}\,dx = \lim_{t \to 0^+} \left[2\sqrt{x}\,\right]_{t}^{4} = 4$$

따라서 특이적분 $\int_{-1}^{4} \frac{1}{\sqrt{|x|}}\,dx$는 수렴하고 그 값은 6이다.