

2016학년도 1학기

# 수학전공 Colloquium

**제 목** Iwasawa theory and the Tate-Shafarevich group

**연 사** 서수길(연세대)

## 초 록

For a number field  $k$  and odd prime  $p$ , we investigate the gap between the condition for a finite cyclic  $p$ -extension  $K/k$  to be  $\mathbb{Z}_p$ -extendable and the condition  $\mathbb{Z}[K:k|p^r\mathbb{Z}$ -extendable for all  $r \geq 0$  where the second condition is said to be cyclic  $p^\infty$ -extendable in this paper.

For the Galois group  $G_S$  of the maximal  $p^\infty$ -ramified extension of  $k$  and the  $p^r$ th roots  $\mu_{p^r}$  of unity, we describe the Shafarevich-Tate group  $\text{III}^1(G_S, \mu_{p^r})$  as the quotient of two subgroups of the idele group of  $k$  such that the numerator is connected to the condition of  $\mathbb{Z}_p$ -extendable and the denominator to the condition of cyclic  $p^\infty$ -extendable.

For a number field  $k$ , the Leopoldt conjecture for  $(k, p)$  can be shown to be equivalent to  $\{|\text{III}^1(G_S, \mu_{p^r})|\}_{r \in \mathbb{N}} < \infty$ , and for almost all odd primes  $p$ , we show under the Leopoldt conjecture that the smallest integer  $p^s$  which makes the sequence  $\{|\text{III}^1(G_S, \mu_{p^r})|\}_{p^r \geq p^s}$  constant is equal to the exponent of the torsion subgroup of the maximal  $p$ -subgroup of the abelianization  $G_S^{ab}$  of  $G_S$ .

In this case, we can evaluate explicitly the residue  $\text{res}_{s=1} \zeta_p(k, s)$  of the  $p$ -adic zeta function  $\zeta_p(k, s)$  at  $s=1$  when  $k$  is totally real.

**일 시** 5월 11일 수요일 5시

**장 소** 5E102